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This technical guide introduces tools and methodologies for simulating light scattering in the Advanced Systems Analysis Program (ASAP®) from Breault Research Organization, Inc. (BRO®). This provides you with a comprehensive guide to command usage, rather than an exhaustive treatise on scatter theory. Therefore, the assumption is that the you are familiar with the fundamental physical concepts of light scattering. Uninitiated readers are strongly encouraged to acquaint themselves with the work of Stover\textsuperscript{1}, Bennett\textsuperscript{2}, Beckmann\textsuperscript{3}, and Harvey\textsuperscript{4} before proceeding.

The first section of this guide, “Scatter Terminology”, introduces concepts and terminology essential to the treatment of scatter. ASAP scatter models are cataloged in the second section, “Types of Scatter Models” on page 15. The third section, “Scatter Methods” on page 48, discusses the supplementary ASAP commands that control the implementation of scatter models. Examples are included throughout to demonstrate command syntax. The last section, “BSDF Fit Utility” on page 60, introduces the ASAP utility for Harvey and polynomial models.

**SCATTER TERMINOLOGY**

**Bi-Directional Scatter Distribution Function (BSDF)**

The specular reflective and transmissive properties of an interface are governed by the Fresnel equations. Based upon incident angle and material properties such as refractive index and absorptance, the Fresnel equations can be used in a ray tracing calculation to apportion flux to transmitted and reflected rays, as well as to determine their respective directions. In an analogous manner, a characteristic function known as the Bi-Directional Scatter Distribution Function (BSDF) describes the angular distribution of radiation scattered from an interface. However, the implementation of BSDF in ASAP is phenomenological, based purely upon mathematical models that describe observed scatter.

characteristics. For this reason, it is sufficient to define BSDF in terms of incident and scattered angles only, with no reliance on material properties.

A formal definition of BSDF is,

$$BSDF = \frac{P_i}{\Omega} = \frac{P_i}{\Omega \cos \theta_s}$$

A practical understanding of BSDF can be obtained by examining the experimental layout shown in Figure 1. This is the basic configuration of a scatterometer, a device typically used to measure scatter properties.

Consider a laser source with power $P_{\text{laser}}$ incident upon a surface at a fixed angle $\theta_0$, with respect to its surface normal. The irradiance at the scattering surface is equal to the laser power $P_{\text{laser}}$ divided by the area $A_{\text{spot}}$ illuminated by the laser. A detector of known dimensions is placed a distance $R$ from the illuminated area and swept through a range of angles $\theta$—most often in the plane of incidence. The radiance $L$, as seen from the detector, is the scattered power $P_{\text{scatter}}$ divided by the area $A_{\text{spot}}$ of the illuminating spot divided by the solid angle $\Omega$ subtended by the detector. The solid angle $\Omega$ is simply the area $A_{\text{detector}}$ divided by the square of the distance $R$ between the illuminated spot and the detector surface. A factor of $\cos \theta$ is necessary to account for the obliquity factor. The obliquity factor leads to the use of the term projected area; that is, radiance is power per solid angle per projected area.
In defining BSDF, it is essential to assume a linear relationship between incident and scattered power. Given this basic assumption, the functional definition of BSDF is the radiance of a surface divided by the irradiance falling upon the surface:

$$\text{BSDF} = \frac{L(\theta, \phi, \theta_o, \phi_o)}{E}$$

where $\theta$ and $\phi$ are the angles measured from and around the surface normal.

**Total Integrated Scatter**

The Total Integrated Scatter (TIS) is defined as the power scattered into the hemisphere above a surface divided by the power incident upon the surface. It is equal to the integral of the BSDF function over the hemisphere multiplied by a cosine obliquity factor:

$$\text{TIS} = \int_0^{\pi/2} \int_0^{2\pi} \text{BSDF}(\theta_o, \theta, \phi_o, \phi) \cos(\theta) \sin(\theta) d\theta d\phi$$

**NOTE** The above expression for TIS is, in general, a function of the incident angles $\theta_o$ and $\phi_o$.

Davies' radar scattering paper from 1954 postulated the following relationship between TIS and surface roughness:

$$\text{TIS} \approx \left( \frac{2\pi \sigma \Delta n}{\lambda} \right)^2$$

where $\sigma$ is the RMS surface roughness, $\Delta n$ is the difference in refractive indices between the two media, and $\lambda$ is the wavelength.

The following list of assumptions must all hold for the equation to be valid:

- The surface is smooth surface ($\lambda >> 2\pi \sigma \Delta n$).
- Most of the light is scattered near the specular direction.
- Scattering originates solely at the top surface, and is not attributable to material inhomogeneity or multilayer coatings.
- The surface is clean.

---

**DIRECT NAMING OF ENTITIES**

Direct naming of entities and BSDF models is not allowed in ASAP. However, this limitation can be circumvented by assigning a number to a descriptive variable definition, as shown in the example script.

```
EDGES (COLD_STOP=1)
  RECT 2 0 2@1

MODELS (GREY_BODY=2)
  LAMBERTIAN 0.5
```
ASAP Scatter Models

Scatter models are entered into the ASAP MODELS database. You create a list of the required BSDF models accompanied by appropriate options. ASAP assigns each MODEL a number in order of entry to be used for referencing purposes. The default number for a MODEL is one more than the highest defined. ASAP allows a maximum of 100 scatter-model definitions.

Once defined, the MODEL scatter properties are applied to individual objects using the following object modifiers. A scattering object can have one or more scatter modifiers.

- **SCATTER MODEL(S) / TOWARDS**: Offers a wide range of model types. Used with importance edges or directions to specify preferential scattering solid angles. Choose number of scattered rays per incident ray.

- **SCATTER RANDOM / TOWARDS**: Specifies a Lambertian scattering surface. Used with importance edges or directions to specify preferential scattering solid angles. Choose number of scattered rays per incident ray.

- **ROUGHNESS MODEL / RANDOM**: Specifies roughness in terms of height variations or deviations in surface normal. One scattered ray per incident ray.

As an example, the following PLANE object has attached to it a scatter model with 50 Å micro-roughness, an autocorrelation length of 20 μm and slope variation of 0.1 radians:

```
WAVELENGTH 500 NM
SURFACE
PLANE 2 0 RECTANGLE 1 2
OBJECT 'SCAT_SURF'
INTERFACE COAT ABSORB AIR AIR
ROUGHNESS 5 2E4 RANDOM 0 0.1 !!!object modifier
```

**SCATTER MODEL OBJECT MODIFIERS**

The scatter model object modifiers in ASAP are described in this section.

**Assigning Scatter Properties to an Object with SCATTER MODEL RANDOM Command**

The **SCATTER MODELS / RANDOM** object modifier assigns scattering properties to an object, and specifies importance area or direction sampling for scattered rays. If reflective m and transmissive m' scatter models are different, specify both. The **RANDOM** option may be used with the **MODEL** option to generate a seed ray; that is, a ray propagating in a direction other than the importance sampling edge, to keep the ray trace going. All rays generated by **RANDOM** have the same flux. To produce scattered rays, **SCATTER MODEL m** must be followed by the **TOWARDS** modifier discussed under the section, “Importance Area
Sampling” on page 54. More diffuse scatter rays directed towards importance areas can be specified with additional lines of input (up to 10 per object). The \texttt{RANDOM} option does not require the modifier \texttt{TOWARDS}, but BRO recommends it since it is more efficient to scatter towards an importance sampled edge.

Using the \texttt{MODELS} command requires non-zero specular reflection and transmission coefficients on the \texttt{INTERFACE} command. If the surface has only transmissive and reflective scatter, use \texttt{INTERFACE 1E-15 1E-15} to eliminate the specular rays while still allowing two-way scatter. While the \texttt{LEVEL} command is in effect, the \texttt{RANDOM} option causes ASAP to generate a set of randomly distributed diffuse scatter rays for each incoming ray on the object. The $r'$ is the additional total hemispherical diffuse reflectivity of the scattering surface. The $r'=1$ corresponds to white Lambertian. The $n$ is the number of rays to be scattered into the hemisphere, centered on the ray intersection point. If both reflective and transmissive (or multiple diffraction orders) scatter rays are generated (see the command, “Setting Maximum Number of Scattering Levels with LEVEL Command” on page 14), the specified scatter can be separately scaled in magnitude for the two components by the given $r$ and $t$ factors. The scatter specified can also be either absolute (\texttt{ABS} referenced to incident power) or relative (\texttt{REL}) to the specular power. The default is \texttt{ABS 1 1} (except for \texttt{SCATTER RMS}, which is \texttt{REL 1 1}).

**Assigning Roughness with the ROUGHNESS Command**

```
ROUGHNESS [ r [ 1 ] ] [ MODEL i RANDOM h [ s [ n ] ] ] [ FRACTION [ f ] ]
```

The \texttt{ROUGHNESS} object modifier assigns roughness in terms of macroscopic random height and surface normal deviations to an object. The $r$ parameter is an RMS micro-roughness in wavelength units. Use it to transfer energy out of the specularly reflected/refracted beams (and usually into any scattered beams) to conserve energy between specular and scattered beams. It does not affect the optical path lengths or directions of the specular beams, nor does it generate any scattered rays. If the autocorrelation length $l$ (also in \texttt{WAVELENGTH} units) is given, the effects of shadowing at high angles of incidence are included. For a \texttt{FRACTION f} (default 1 or given \texttt{MODEL TIS}) of the incident rays, \texttt{RANDOM} or \texttt{MODEL} assigns a macroscopic height variation $h$ (default value 0). This is a Gaussian distribution with an RMS value entered in \texttt{WAVELENGTH} units and slope error $s$ (default value 0) in radians to the surface(s) of the object. The random height variations affect only the position of a point on the object and, therefore, the optical path lengths of any reflected or refracted beams, while the slope errors affect only the normal (and thus the beam directions).

The \texttt{RANDOM} normal deviations $s$, entered as an RMS value in radians, affect only the surface normal and, therefore, the beam directions. The maximum allowable normal deviation is 0.2 radians (ASAP PRO). If $s$ exceeds about one fifth (0.2 radians), unexpected ray trace results may occur at the surface. That is, wrong side warnings may be generated because, for example, a ray may randomly reflect into the surface. For near normal
incidence rays, if the RMS slope error exceeds about one fifth (0.2), unexpected ray trace results may occur at the surface; for example, "wrong side" warnings, because a ray may randomly reflect or refract into the surface.

For near-grazing incidence rays, the RMS slope may have to be much smaller than this to avoid these ray trace errors. Both errors are, by default, generated according to an approximately Gaussian-normal distribution (type 2 below). However, the slope error distribution function can be any one of the following:

\[\text{To be more precise, if the randomly unperturbed normal points along the z axis, the components of the randomly perturbed normal are:}\]

\[
\left[ s \cos(u), s \sin(u), \sqrt{1-s^2} \right]; \quad -1 < s < 1 \quad \text{and} \quad -\pi < u < \pi
\]

where \(u\) is a uniformly distributed random number, and \(s\) is a random number determined by the given slope distribution (RMS and probability function). Note the maximum absolute slope error is always limited to one; that is, 45 degrees. Alternatively, the surface slopes may be randomized so that the shape of the normal incidence reflected pattern matches that of the BRDF specified by the scattering \textit{MODEL i}.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{m} & \textbf{Slope distribution} & (Maximum/RMS)^2 & Equivalent \\
\hline
-3 & Two deltas & 1 & RAN(-15) \\
\hline
-2 & Lambertian & & \\
\hline
-1 & Ramp & 2 & RAN(-1) \\
\hline
0 & Uniform & 3 & RAN(0) \\
\hline
1 & Triangular & 6 & RAN(2) \\
\hline
2 & Gaussian-like & 9 & RAN(3) \\
\hline
3 & Cosine & 5 & RAN(1) \\
\hline
4 & Near-Gaussian & 15 & RAN(5) \\
\hline
5 & Gaussian & 2 \ln(2^32) & RAN(15) \\
\hline
\end{tabular}
\caption{Slope error distribution function}
\end{table}
Setting Maximum Number of Scattering Levels with LEVEL Command

```
LEVEL [ n ] [ OFF ] [ c ]
LEVEL ALL
```

The LEVEL command sets the maximum number of scattering levels. This command can be applied globally or on an object-by-object basis. LEVEL controls scattering in much the same way that SPLIT controls specular ray splitting. The integer n is the maximum number of scattering levels allowed for random diffuse rays. For example, LEVEL 1 tells ASAP to split the parent rays and not to split the child scatter rays. LEVEL 2 allows only the parent and first generation child rays to split. The parameter c (default 1E-12) sets the fractional scattered energy relative to the incident ray, below which no scattered ray is generated. The OFF option turns off all scatter. Normally, scattered rays are not generated for any specular child rays. If ALL is specified, scattered rays are generated for each specular child ray (including multiple diffraction orders, extraordinary rays, and so on), so that ‘bi-directional’ scatter can occur at a partially reflecting interface. The following example illustrates the use of LEVEL ALL in specifying bi-directional scatter. A single ray incident at 45° scatters 20 rays in both forward and backward directions into a 0.2 radian cone angle centered on the surface normal as specified by the TOWARDS command.

```
SYSTEM NEW
RESET
UNITS MM

MEDIA
  1.5 'GLASS'

MODELS
  LAMBERTIAN .3
  LAMBERTIAN .7
RETURN

FRESNEL AVE
MISSED ARROWS 1
LEVEL ALL

SURFACE
  PLANE Y 0 RECT 1
OBJECT 'FRONT_SIDE'
INTERFACE 1E-15 1E-15 AIR GLASS
FRESNEL OFF
SCATTER MODEL 1
TOWARDS SPEC 20 0.2 0

GRID ELLIP Y 0 -45 1 201
SOURCE DIRECTION 0 -1 TAN [45]
MOVE BY Y 2

WINDOW Y 2
PLOT OVERLAY
TRACE PLOT
```
TYPES OF SCATTER MODELS

The accuracy of any scatter calculation depends upon how well a given model describes an illuminated surface. While the mathematical constructs used to define scatter models range from simple to rather complex, experience has shown that a relatively small set of model types cover virtually all practical physical situations. These models are integrated into ASAP, and are easily invoked with the appropriate key word.

Isotropic Scatter Models

The following models are intrinsically isotropic; that is, they apply to surfaces whose scatter properties at any angle of incidence are independent of object rotation about the normal.

LAMBERTIAN
HARVEY
KCORRELATION
ABG
BINOMIAL/TRINOMIAL/POLYNOMIAL
NONLINEAR
RMS
PHYSICAL
PARTICLES
VOLUME [MIE]
VCAVITY
USERBSDF
BSDFDATA/RAWDATA
SUM

General mathematical models of isotropic-surface scattering must be symmetric with regard to the plane of incidence and surface normal; that is, scatter pattern does not change when the surface element is rotated about its normal. If the BSDF is only a function of the following variables, then these symmetry properties are guaranteed.

\[ U = \alpha_s^2 + \beta_s^2 < 1 \quad V = \beta_s \beta_o \quad W = \beta_o^2 < 1, \]

where \((\alpha_s, \beta_s) = (\sin \phi_s \sin \theta_s, \cos \phi_s \sin \theta_s)\) are the direction cosines coordinates of the scatter direction and \((0, \beta_o) = (0, \sin \theta_o)\) is the specular direction (see Figure 1). The \(\phi_s\) and \(\theta_s\) also correspond to the scatter angles around (azimuthal) and from (polar) the normal, and \(\theta_o\) is the specular angle from the normal. Furthermore, note that the distance squared (in terms of direction cosines) from specular is given by:

\[ T = \alpha_s^2 + (\beta_s - \beta_o)^2 = U - 2V + W < 4. \]
Any physical BSDF must obey reciprocity; that is, be the same when the scatter (U) and specular (W) directions are interchanged. Therefore, the BSDF dependence on U should be identical to its dependence on W. Unfortunately, due to errors, there are a lot of models and even measured data that violate reciprocity.

**LAMBERTIAN (ISOTROPIC)**

The **LAMBERTIAN** model represents an idealized surface whose radiance or luminance is independent of direction. It is useful in simulating emission, and provides a reasonable approximation to a large range of diffuse surfaces, such as flat finish paints and Spectralon™—provided the angle of incidence is not too large. All materials tend to scatter more light around the specular direction, thereby violating the Lambertian approximation at large angles of incidence.

The BSDF of a Lambertian surface is a constant equal to its total hemispherical diffuse reflectivity $r$ divided by $\pi$. As a result, the TIS of a Lambertian surface is also a constant. **LAMBERTIAN** is the only scatter model whose TIS is independent of incident angle.

The **LAMBERTIAN** model also finds use as an aid to efficient ray tracing. Cote and Tesar\(^6\) demonstrate how this model, in concert with importance area sampling, economize tracing in an imaging application. The source is created with **EMITTING DATA**, using a display file (*.dis) produced by the BMP2DIS utility from a bitmap. When traced, rays are scattered from a dummy surface in close proximity to the source **TOWARDS** and importance **EDGE** located at the entrance aperture of a camera lens.

**TIP** An ASAP feature invokes **BMP2DIS** directly from the **DISPLAY** command input, thus eliminating the need to launch the conversion utility from a Windows command prompt (>). A single ASAP command converts a bitmap (*.bmp) to a display file (*.dis) containing RGB (RED GREEN BLUE) or monochrome (MONO) component data.

\[\text{DISPLAY } \text{filename}.BMP \quad \text{RED} \quad \text{GREEN} \quad \text{BLUE} \quad \text{MONO}\]

---

6. Cote and Tesar
SCATTERING IN ASAP
Types of Scatter Models

HARVEY (ISOTROPIC)

The Harvey model was developed by J. E. Harvey to predict scattering characteristics of optically polished surfaces. The essence of the Harvey model states that the scatter behavior of applicable surface types does not depend on the incidence angle alone, but rather upon the angular difference between the scattered and specular rays. In other words, the Harvey model is shift invariant with respect to incident angle. In mathematical terms,

$$BSDF_{Harvey} = b \left(100 \left| \sin \theta_{scat} - \sin \theta_{spec} \right| \right)^s = b \left(100 \left| \beta_{scat} - \beta_{spec} \right| \right)^s$$

where $b$ is a constant corresponding to the BSDF at 0.01 radians (0.573 degrees) from specular, $s$ is the asymptotic fall-off with angle (typically between -1 and -2.5) and $\beta = \sin \theta$. $\theta_r$ and $\theta_s$ are the polar angles of incident and scatter rays, respectively.

A limitation of the Harvey model arises from the existence of a singularity in the BSDF when the specular and scatter angles are equal. Consequently, the model was adapted to include a shoulder parameter $\ell$ (in radians) that rolls the BSDF over, thereby insuring conservation of energy near specular angles and a finite TIS. This modified Harvey scatter function takes the form,

$$BSDF_{Modified 
Harvey} = b_0 \left[ 1 + \left( \frac{\sin \theta}{\ell} \right)^2 \right]^{\ell/2}$$

where $\theta$ = the scatter angle from specular

$b_0$ = the BSDF at $\theta = 0$

$s$ = the slope of the BSDF on a log-log plot for $\sin \theta/\ell >> 1$, and

$\ell$ = the shoulder of the BSDF curve.

Note: the straight-line extrapolated intercept $b_0$ is related to the original $b$ by $b_0 = b(100\ell)^s$

See Figure 2“Log plot of Harvey and modified Harvey showing $b$, $b_0$, and $l$” on page 18.
The total integrated scatter of the modified Harvey command is:

\[
TIS = 2\pi b \frac{100^s}{s + 2} \left[ \left(1 + t^2\right)^{\frac{s+2}{2}} - \left(t^2\right)^{\frac{s+2}{2}} \right] \quad \text{where } s \neq -2
\]

\[
TIS = 2\pi b \frac{(100t)^s}{2} t^2 \ln\left(1 + \frac{1}{t^2}\right) \quad \text{where } s = -2
\]

The Harvey model is wavelength dependent. Note that wavelength scaling is valid only for clean, smooth surfaces (RMS roughness \(\sigma \ll \lambda\)) over a limited range of \(\lambda\). The model parameters scale as follows:

\[
b(\lambda) = b(\lambda_0)\left(\frac{\lambda_0}{\lambda}\right)^{s+5}
\]

\[
s(\lambda) = s(\lambda_0)
\]

\[
\ell(\lambda) = \ell(\lambda_0)\left(\frac{\lambda}{\lambda_0}\right)
\]
The reference wavelength $\lambda_o$ can be specified on the command line with the option $w$. Set $w$ to zero to turn off wavelength scaling.

**NOTE** Care should be exercised in applying the Harvey model to refractive optical elements. Since measurement of the BTDF (Bi-Directional Transmittance Distribution Function) of a transmissive element necessarily takes into account scatter from both surfaces of a witness sample, the composite values for $b$ and $s$ need to be adjusted.

For BTDF data fit to the Harvey functional form,

$$\text{BTDF} = b \{100 |\beta - \beta_o|\}^i,$$

and the rare-to-dense interface **MODEL** becomes

$$\text{BTDF} = \frac{1}{2} b \ n^{2+s} \ \{100 |\beta - \beta_o|\}^i,$$

and the dense-to-rare interface **MODEL** should be scaled according to

$$\text{BTDF} = \frac{1}{2} b \{100 |\beta - \beta_o|\}^i.$$

This scaling is a direct consequence of the Snell’s Law relationship between incident and refracted angles.

The $m$ and $n$ are two additional invariance parameters postulated for rough surfaces. If

$$\text{BSDF} = h f \left(\frac{B - B_o}{l}\right)^i f \{x\} = \sqrt{1 + x^2}$$

for $m=0$ and $n=0$, then

$$g = \text{BSDF} = h f \left(\frac{B - B_o}{l \ g^n}\right)^i g = \frac{C + C_o}{2}$$

where $B$, $C$ are the sine, cosine of the scatter angle from normal and $B_o$, $C_o$ are the sine, cosine of the specular angle. For typical rough surfaces, $m$ is approximately 2 and $n$ is approximately 1.

Figures 3 and 4 contrast the ‘smooth surface’ ($m=0$, $n=0$) and ‘rough surface’ ($m=2$, $n=1$) Harvey models. Two distinct differences become immediately evident in these log(BSDF) versus angle plots. First, unlike the ‘smooth’ surface model, the BSDF of the ‘rough’ surface roll upwards for large scatter angles. Secondly, with the ‘smooth’ surface, TIS falls with increasing angle of incidence while with the ‘rough’ surface, TIS increases.

Figure 3 shows a smooth surface Harvey model, and Figure 4 shows a rough surface.
Figure 3 Smooth surface Harvey model \( (b_o=0.1, s=-1.2, \ell=0.02, m=0, n=0) \)
Figure 4 Rough surface Harvey model ($b_o=0.1$, $s=-1.2$, $\ell=0.02$, $m=2$, $n=1$)
KCORRELATION (ISOTROPIC)

The K-Correlation scatter model is used to simulate scatter from smooth surfaces, whose power spectral densities can be fit to an algebraic form. This is the full form of K-Correlation:

$$BSDF(\beta, s \neq 2) = \frac{2\pi(dn)^2RB^2}{\lambda^4} \frac{\sigma^2(\lambda)(s-2)}{[1 - [1 + B^2/\lambda^2]^{1-s/2}]} \frac{\cos(\theta_i)\cos(\theta_s)}{[1 + B^2\beta^2/\lambda^2]^{s/2}}$$

$$BSDF(\beta, s = 2) = \frac{4\pi(dn)^2RB^2}{\lambda^4} \frac{\sigma^2(\lambda)}{\ln(1 + B^2/\lambda^2)} \frac{\cos(\theta_i)\cos(\theta_s)}{[1 + B^2\beta^2/\lambda^2]}$$

**NOTE** The scaling laws that are used for total effective roughness as a function of wavelength have not been implemented in ASAP as of 2012 V1R1. However, you can use them outside of ASAP:

$$\frac{\sigma^2_{s=2}(\lambda_2)}{\sigma^2_{s=2}(\lambda_1)} = \frac{1 - [1 + B^2/\lambda_2^2]^{1-s/2}}{1 - [1 + B^2/\lambda_1^2]^{1-s/2}}$$

$$\frac{\sigma^2_{s=2}(\lambda_2)}{\sigma^2_{s=2}(\lambda_1)} = \frac{\ln[1 + B^2/\lambda_2^2]}{\ln[1 + B^2/\lambda_1^2]}$$

The parameter $r$ is for surface specular reflectivity, ($R$). The parameter $\text{rms}$ is the total effective RMS (root mean squared) surface roughness over frequencies from 0 to $1/\lambda$, in units of $\mu$m, ($\sigma(\lambda)$). The parameter $b$ is related to the surface correlation length ($L_c$), in units of $\mu$m, ($\beta$). The parameter $w$ is the measurement wavelength, in units of $\mu$m, ($\lambda$). The parameter $dn$ is the change in index of refraction at the surface, ($dn$). The parameter $s$ is the slope of the Log-log BSDF plot ($S_2$ and BRDF function) at large spatial frequencies.
The remaining parameters and definitions are listed in Table 2, some being intrinsic to ASAP.\textsuperscript{7}

\textit{Table 2. With the K-Correlation model, the PSD and BRDF are completely defined by:}

<table>
<thead>
<tr>
<th>Formal</th>
<th>ASAP</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>w</td>
<td>Measurement wavelength ((\mu m))</td>
</tr>
<tr>
<td>s</td>
<td>s</td>
<td>Slope of (S_2) and BRDF function at large spatial frequencies</td>
</tr>
<tr>
<td>B</td>
<td>b</td>
<td>(2\pi) times surface correlation length (L_c) ((\mu m))</td>
</tr>
<tr>
<td>(\sigma(\lambda))</td>
<td>(\text{rms})</td>
<td>Total effective rms roughness over frequencies from 0 to ((\mu m))</td>
</tr>
<tr>
<td>R</td>
<td>r</td>
<td>Surface specular reflectivity</td>
</tr>
<tr>
<td>(dn)</td>
<td>(\text{dn})</td>
<td>Change in index of refraction at the surface (=2 for mirrors)</td>
</tr>
<tr>
<td>(S_1)</td>
<td></td>
<td>1-dimensional surface PSD ((\mu m^3))</td>
</tr>
<tr>
<td>(S_2)</td>
<td></td>
<td>2-dimensional surface PSD ((\mu m^4))</td>
</tr>
<tr>
<td>(f)</td>
<td></td>
<td>Spatial frequency on the surface ((\mu m^{-1}))</td>
</tr>
<tr>
<td>(\theta_i)</td>
<td></td>
<td>Angle of incidence, relative to surface normal</td>
</tr>
<tr>
<td>(\theta_s)</td>
<td></td>
<td>Scatter angle, relative to surface normal</td>
</tr>
<tr>
<td>(\beta - \beta_0)</td>
<td></td>
<td>Sine of the Scatter angle minus sine of specular reflection angle</td>
</tr>
</tbody>
</table>

Regarding K-Correlation or ABC power spectral density (PSD) and BRDF surface scatter modeling, Eugene Church and John Stover have published descriptions of its use for smooth surfaces. The K-correlation model “provides the capability to model isotropic scattering surfaces that are not fractal; that is, they do not continue at the same log-log slope across all spatial frequencies”, such as with Harvey models. “Many surfaces exhibit roll-off of amplitude at small spatial frequencies (large periods)... This roll-off is determined by the $\beta$ parameter” of the K-Correlation model...The use of this length enables modeling of both conventional and fractal surfaces with a single mathematical tool.”

See Figure 5 and 6.\footnote{8}

---

\textbf{Figure 5} K-Correlation BRDF provides an inherent small-angle roll-off

---

\footnote{The use of K-Correlation BRDF can be found in the work of M. Dittman, Proc. SPIE, Vol. 6291, 62910R, (2006).}

\footnote{Ibid.}
Figure 6 BRDF variation with B shows conservation of energy.
SCATTERING IN ASAP
Types of Scatter Models

ABG (ISOTROPIC)

The ABg scattering model is similar to the Harvey model, in that it is empirically derived from measurement, and is widely used to model scatter that is created by random isotropic surface roughness. The ABg model is a method for defining the Bidirectional Scattering Distribution Function (BSDF). This scattering model is effective when scattering is mainly due to random isotropic surface roughness, and the scale of the roughness is small compared to the wavelength of light being scattered, which is generally valid for polished optical surfaces.

When the BSDF is plotted as a function of $x$, the resulting data is fit to a function of the form:

$$BSDF(\beta - \beta_0) = \frac{A}{(B + |\beta - \beta_0|^g)}$$

$$\beta = \sin(\theta_{\text{scatter}})$$

$$\beta_0 = \sin(\theta_{\text{specular}})$$

where

$$A \geq 0, B \geq 0$$

Coefficient restrictions: $A$ and $B$ must be greater than or equal to 0. If $A$ is zero, no scattering occurs. If $g$ is zero, the BSDF is constant in direction cosine space:

$$BSDF(\beta - \beta_0) = \frac{A}{(B + 1)} = \frac{TIS}{\pi}$$

$$A = TIS$$

$$B = \pi - 1$$

and the resulting scattering is Lambertian. If $A$, $B$, and $g$ are set to provide a relatively flat BSDF curve, using a Lambertian scatter model is preferable.
SCATTERING IN ASAP
Types of Scatter Models

The parameter $a$ is the specular peak numerator, $(A)$, where $a/b$ is the specular peak. The parameter $b$ is related to the surface correlation length ($L_c$), in units of $\mu m$, $(B)$. The parameter $g$ is a scattering quantitative and directional term that may be positive or negative, but is typically between 0 and 3.

Table 3. With the $ABg$ model, BSDF is defined by:

<table>
<thead>
<tr>
<th>Formal</th>
<th>ASAP</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$a$</td>
<td>Specular peak numerator</td>
</tr>
<tr>
<td>$B$</td>
<td>$b$</td>
<td>Specular peak denominator $\pi$ times surface correlation length $L_c$ ($\mu m$)</td>
</tr>
<tr>
<td>$g$</td>
<td>$g$</td>
<td>A scattering quantitative and directional term that may be positive or negative, but is typically between 0 and 3.</td>
</tr>
</tbody>
</table>

POLYNOMIAL/TRINOMIAL/BINOMIAL (ISOTROPIC)

This command has two syntaxes. The first syntax is:

```
POLYNOMIAL n m [ l [ l' [ d ] ] ] .... options
TRINOMIAL
BINOMIAL
```

The POLYNOMIAL/TRINOMIAL/BINOMIAL command creates a scatter model based upon a polynomial expression involving direction cosines. A common usage of this scatter model comes in simulating scatter behavior of diffuse black paints. This scatter behavior is best recognized on plots of the log(BSDF) versus log($\beta_s$-$\beta_r$) as a line that gradually branches up and down as the abscissa increases (see Figure 5). This branching is characteristic of materials that show different backward and forward scatter properties, depending upon whether the direction of the scattered light follows the direction of the incident light or returns in the opposite direction.

In ASAP, scattering is described by a general polynomial of three symmetry variables U, V, and W. This formalism ensures that scattering from a surface is symmetric with regard to the plane of incidence and surface normal. In other words, the following formalism only applies to isotropic surfaces and symmetry properties are guaranteed if the BSDF is only a function of the following variables.
The three models are defined by the following equations:

**POLYNOMIAL**

\[ m > 0: \]
\[
\log(\text{BSDF}) = \sum_{k=0}^{n} \left\{ \sum_{i=0}^{m} \sum_{j=0}^{i} c_{ijk} \left( U^j W^j + U^j W^i \right) + \sum_{i=l}^{l'} c_{ik} \log \left( 1 + d^i T \right) \right\} V^k
\]

\[ m < 0: \]
\[
\log(\text{BSDF}) = \sum_{k=0}^{n} \left\{ \sum_{i=0}^{m} \sum_{j=0}^{i} c_{ijk} \left( U^j W^j + U^j W^i \right) + \sum_{i=l}^{l'} c_{ik} \log \left( 1 + d^i T \right) \right\} V^k
\]

**TRINOMIAL**

\[ m > 0 \]
\[
\log(\text{BSDF}) = \sum_{k=0}^{n} \left\{ \sum_{i=0}^{m} \sum_{j=0}^{i} c_{ijk} U^j W^j + \frac{1}{2} \sum_{i=l}^{l'} c_{ik} \log \left( 1 + d^i T \right) \right\} V^k
\]

\[ m < 0 \]
\[
\log(\text{BSDF}) = \sum_{k=0}^{n} \left\{ \sum_{i=0}^{m} \sum_{j=0}^{i} c_{ijk} U^j W^j + \frac{1}{2} \sum_{i=l}^{l'} c_{ik} \log \left( 1 + d^i T \right) \right\} V^k
\]

**BINOMIAL**

\[ m > 0 \]
\[
\log(\text{BSDF}) = \sum_{k=0}^{n} \left\{ \sum_{i=0}^{m} c_{ik} \left( \sqrt{T} \right)^i + \frac{1}{2} \sum_{i=l}^{l'} c_{ik} \log \left( 1 + d^i T \right) \right\} V^k
\]

\[ m < 0 \]
\[
\log(\text{BSDF}) = \sum_{k=0}^{n} \left\{ \sum_{i=0}^{m} c_{ik} \left( \sqrt{T} \right)^i + \frac{1}{2} \sum_{i=l}^{l'} c_{ik} \log \left( 1 + d^i T \right) \right\} V^k
\]

where \( m, n, 1, 1' \) are summing indices, \( d \) is a logarithmic coefficient, and \( c, c', c'' \ldots \) are the applicable polynomial coefficients.

The figure below is a plot of \( \log(\text{BSDF}) \) versus \( \log(\beta - \beta_0) \) for Aeroglaze Z306, a black paint used in the defense industry to coat opto-mechanical parts.
The second syntax of this command is:

```
POLYNOMIAL n m [ l [ l' [ d ] ] ] FIT ....options
TRINOMIAL
BINOMIAL
```

Measured data can also be **FIT** to these model types using either a Lorentzian or Single-Value-Decomposition (SVD) algorithm. The user need only decide the order of the polynomial; that is, values for *m* and *n*. A list of the best-fit coefficients *c* *c'* *c''*... can be displayed in the Command Output window by means of the **PRINT** command as shown in Figure 8.
**NONLINEAR (ISOTROPIC)**

This command has two syntaxes. The first syntax is:

\[
\text{NONLINEAR n \ a \ b \ c \ d \ e \ [\ a' \ b' \ c' \ d' \ e' \ [\ a'' \ b'' \ c'' \ d'' \ e'' \ ...] \ ] \ .... \ options}
\]

\text{NONLINEAR} is a scatter model that combines the Phong (broad peak) and Harvey (sharp peak) models and, as such, is applicable to both smooth and rough surfaces. Two separate command formats allow you to enter directly either the appropriate coefficients or measured data. Data is \text{FIT} to the following equation by an iterative, non-linear, damped, least-squares algorithm.

The BSDF of a \text{NONLINEAR} scatter model is defined by the formula,

\[
BSDF = \sum_{i=1}^{N} \left\{ a_i \left( U + W \right) + b_i V + c_i \sqrt{1-U} \sqrt{1-W} + d_i \right\}^{e_i}
\]

where \( U, V, \) and \( W \) are defined under \text{POLYNOMIAL}. The \( e \) exponents do not need to be integer or positive. The total number of parameters must be less than \( (\leq) \) or equal to \( (=) \) 285; that is, \( N \) less than \( (\leq) \) or equal to \( (\leq) \) 57 quintuples. Five terms or fewer is usually sufficient for most surfaces. If the quantity in braces \{\} is less than zero, the term is set to zero.

Special cases of the general \text{NONLINEAR} model are listed in the following table.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Exponent Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward Harvey</td>
<td>( a_i &gt; 1 ) ( b_i = -2a_i ) ( c_i = 0 ) ( d_i &gt; 0 ) ( e_i &lt; 0 )</td>
</tr>
<tr>
<td>Retro Harvey</td>
<td>( a_i &gt; 1 ) ( b_i = 2a_i ) ( c_i = 0 ) ( d_i &gt; 0 ) ( e_i &lt; 0 )</td>
</tr>
<tr>
<td>Lambertian</td>
<td>( a_i = 0 ) ( b_i = 0 ) ( c_i = 0 ) ( d_i &gt; 0 ) ( e_i = 1 )</td>
</tr>
<tr>
<td>Forward Phong</td>
<td>( a_i = 0 ) ( b_i = c_i ) ( c_i &gt; 0 ) ( d_i = 0 ) ( e_i &gt; 1 )</td>
</tr>
<tr>
<td>Retro Phong</td>
<td>( a_i = 0 ) ( b_i = -c_i ) ( c_i &gt; 0 ) ( d_i = 0 ) ( e_i &gt; 1 )</td>
</tr>
</tbody>
</table>
The second syntax for this command is:

```
NONLINEAR FIT [ n ] ....options
FRAC e e' e'' ....
data...
```

An alternative syntax allows you to `FIT` measured data to the above polynomial expression. When attempting a `FIT` to data, use the data format detailed in “BSDF Fit Utility” on page 60. The `ANGLES` must be included when specifying spherical angle coordinates. The `LOG` option can be used when BSDF data values are provided in common logarithmic form. Since the `NONLINEAR` model is not defined in logarithm space (like the `POLYNOMIAL` model), the `FIT` may have a difficult time accurately reproducing any BSDF with a high dynamic range. Optionally, the fractional (`FRAC`) error at each data point can be used instead of the absolute error. This option has about the same effect as fitting in logarithm space. The nature of an iterative, non-linear, damped, least-squares algorithm may converge to a local minimal rather than a desired global minimum. An off-line, global optimization technique (for example, simulated annealing) could be used if the ASAP fit is unsatisfactory.

**RMS (ISOTROPIC)**

```
RMS r l s ...options
```

The RMS model is useful in the case where you have RMS height variation $r$, autocorrelation length $l$ and asymptotic fall-off $s$ ($0=$Lambertian, otherwise typically between -1 and -2.5) with angle from specular. The $\beta-\beta_0$ shoulder point is the ray's wavelength divided by $l$. This is an approximate (violates reciprocity) theoretical model primarily for smooth surfaces ($r$ much less than a wavelength and $l$ much greater than a wavelength), but it otherwise behaves well (although it may not represent any actual rough surface). The BSDF varies with wavelength, incidence direction, scatter direction, and the specific object's `INTERFACE` properties. It also automatically conserves energy (within standard statistical variations), if a `ROUGHNESS r l` command is also assigned to the object.

**PHYSICAL (ISOTROPIC)**

```
PHYSICAL s l [ r ] [ GAUSSIAN ] ....options
coat
```

Much like the RMS model, PHYSICAL is defined in terms of the statistical properties of surface height variation. The $s$ parameter is the slope of the BRDF when plotted against $\sin(\theta) - \sin(\theta_0)$ on a log plot (see “HARVEY (Isotropic)” on page 17). The auto-correlation length $l$ is based upon a fractal or Gaussian-shaped Power Spectral Density Function.
SCATTERING IN ASAP

Types of Scatter Models

(PSDF). You specify either the normal-incidence reflectivity \( r \) or a polychromatic coating name. The coating default is the object specular reflectivity, which is defined with an INTERFACE command. PHYSICAL is a comprehensive physical model that exhibits reasonable behavior—even for rough surfaces at grazing incidence.

PARTICLES (ISOTROPIC)

\[
\text{PARTICLES} \quad g \quad q \quad [ \quad p \quad ] \quad f
\]

\[
\text{VOLUME} \quad \text{MIE} \quad m \quad a \quad a' \quad f \quad [ \quad fcn \quad n \quad c \quad c' \quad \ldots \quad ]
\]

ASAP provides commands that simulate particulate scatter as described by either the Henyey-Greenstein or Mie theories. A PARTICLE model is applied to an object using the SCATTER MODEL modifier. A VOLUME scatter model is applied to a media defined in the MEDIA database using a SCATTER designator.

For these particulate scatter models, ASAP changes the trajectory and flux of each ray as it traces.

**NOTE** Run times can become substantial in the case of a VOLUME calculation, due to the arbitrarily large number of possible intersections.

The Henyey-Greenstein model includes a directional parameter \( g \) that varies from perfect backscatter (-1) to isotropic (0) to perfect forward scatter (+1). The equation and graph in “Figure 9” indicates how this scattering probability varies with the parameter \( g \).

\[
p(\theta) = \frac{1}{4\pi} \frac{1-g^2}{\left(1+g^2-2g\cos(\theta)\right)^{3/2}}
\]

**Figure 9 Reference:** L. G. Henyey, J. L. Greenstein. Diffuse radiation in the galaxy. Astrophysical Journal, 93:70-83, 1941
The parameter \( q \) is the scatter efficiency. The \( p \) allows for optional entry of absorption efficiency per particle. If used, it should immediately follow the scatter efficiency value \( q \) with only a backward tick to separate the two numbers. Either number is typically near or less than one. Entering \( q0 \) is the same as entering \( q \) by itself. The \( f \) option is equivalent to the number of particles-per-unit-area times the average particle cross-sectional area. Therefore, it is usually a small number much less than one.

For particles in a **volume**, \( f \) is the overall fractional area obscuration per unit length, and is equivalent to the number of particles per-unit-volume times the average particle cross-sectional area. Whether the scattering is from real particles or something else, only the product of \( q \) and \( f \) is important, since it is just equal to the standard “extinction” coefficient. Also, whether the scattering is from actual real particles or something else, only the product of \( q+p \) and \( f \) really matters since this is just equal to the standard "extinction" coefficient (\( q \) times \( f \) is inversely proportional to the mean free path length).

Two other models for single sphere scattering are available:

1. An exact, time-consuming MIE calculation with particle refractive indices specified by **MEDIA** \( m \).

2. A fast approximation for large (radii much greater than a wavelength), opaque (indices much greater than one), rough (white Gaussian statistics) spheres with intrinsic surface reflectivity \( r \) and **RMS** surface roughness \( s \) (relative to the sphere radius).

In both cases, the particle radii lie between \( a \) to \( a' \) (in wavelength units) corresponding to the \( e^{-2} \) points of a default Gaussian-normal size distribution.
The following three Examples illustrate methods of implementing various volume scattering models in ASAP.

**PARTICLES Example 1**
Construct a MIE model of polystyrene spheres immersed in water and having a concentration of $10^6$ spheres per cubic millimeter. Sphere radii range from 0.3 to 0.5µm.

```
UNITS MN
WAVELENGTHS (VL=0.632) MN

UCONC=1E6
PRAD1=0.3E-3 PRAD2=0.5E-3 ??Particle radius in system unit MN
FOBS=PI*((PRAD1+PRAD2)/2)^2*UCONC

MEDIA 1.42 'POLY_SPHERE'
1.33 'WATER'
RETURN

IMMERSE WATER
MODELS
VOLUME MIE POLY_SPHERE (PRAD1*1E3) (PRAD2*1E3) (FOBS)

MEDIA 1.33 SCATTER 1 'WATER_SPHERE'
```

The polystyrene and water media must be defined prior to creating the scatter model. Apply the scatter model to the immersion medium after setting the **MODEL** type and parameters.

**PARTICLES Example 2**
Construct a backscattering Henyey-Greenstein model for application to a mirrored surface. Give the model a per-particle scattering efficiency of 0.9, a particle density $\delta$ of 100 mm$^{-2}$, and particle cross section area $\eta$ of $2\times10^{-6}$ mm$^2$.

The overall fractional obscuration is $\delta \eta$. Therefore, the model is constructed as follows:

```
UNITS MN
WAVELENGTHS 500 MN

SCAT_EFF=0.9
DLTA=1E2 NU=2E-6

MODELS
PARTICLES -1 (SCAT_EFF) (DLTA*NU) PLOT 'HG_MODEL'
RETURN
```

**PARTICLES Example 3**
Construct an MIE scatter model, based upon MIL-STD 1246C for visible wavelengths. Let the particles range in size from 1- to 100µm with a fractional obscuration of $3\times10^{-3}$ corresponding to a cleanliness level of 500.
After defining a complex particle index at the appropriate wavelength, a function is constructed based upon MID-STD 1246C and equations from Spyak and Wolfe. The \( \text{FCN} \) function first defines the actual particle diameter \( S \) from the normalized diameter via the \( \_ \) variable. The smallest and largest particle diameters are passed to \( \text{FCN} \) by the particle model in \( \_1 \) and \( \_2 \). The contamination level is passed in \( \_3 \), which must be compatible with the fractional area coverage (\text{FRAC}) to get the correct answer.

**NOTE** The cleanliness level, CLEVEL and fractional obscuration, FRACOBS are dependent variables, but are not treated as such in this example. To obtain a correct result, the value of the fractional area coverage must be set to a value that corresponds to the desired cleanliness level.

```
UNITS   MN
WAVELENGTHS  0.632 Um

MEDIA
  1.53`0.0007 'CONTAM'

\$FCN MSTD  S=(\_2`\_1)*+:\_2+\_1),
  N=10^((0.926*(\text{LOG}[\_3]^2-L\text{OG}[S]^2)),
  N*2*.926*L\text{OG}[S]/S
SNMIN=1  \#MINIMUM PARTICLE DIAMETER
SMAX=100 \#MAXIMUM PARTICLE DIAMETER
FRACOBS=3E-3  \#FRACTIONAL OBSCURATION
CLEVEL=500  \#CLEANLINESS LEVEL
NSAMP=50 \#NUMBER OF INTEGRATION SAMPLES

PIXELS  501
MODEL
  PARTICLES MIE CONTAM SNMIN/2 SMAX/2 (FRACOBS) MSTD (NSAMP),
  SNMIN/2 SMAX/2 (CLEVEL) PLOT 0
RETURN
```

10. Spyak and Wolfe
SCATTERING IN ASAP

Types of Scatter Models

VCAVITY (ISOTROPIC)

The VCAVITY command simulates scatter from a rough surface modeled as a random collection of v-cavities. The cavity slopes are assumed to follow a Gaussian-normal distribution with zero mean. The model has both specular and diffuse components. The specular component can be specified by a reflectance \( r \) or a COATING \( k \). The diffuse component has a reflectance \( r' \) accompanied by either slope \( s \) or height variations \( h \). This virtual-surface model has become obsolete with the advent of ever-faster computers. Its origin is that of a convenient shortcut for serrated surfaces.

USERBSDF (ISOTROPIC)

The USERBSDF command is used to specify a user-defined scatter model based upon a prior $FCN definition. The $FCN variables \(_1\), \(_2\) and \(_3\) are assigned to isotropic surface symmetry variables \( U \), \( V \), and \( W \) as described under POLYNOMIAL. Additional coefficients \( c, c', c'' \ldots \) (up to 63 coefficients) are passed as variables \(_4\), \(_5\) \ldots \(_66\) for use in the $FCN definition.

USERBSDF Example 1

Create a BSDF model that varies as the cosine of the angle from specular raised to some arbitrary power.

\[
$FCN FONG \quad C = 2 + \text{SORT}[(1 - _1) \times (1 - _3)], \quad 4 \times (C > 0)^{-5} + _6
\]

MODEL

\[
\text{USERBSDF } FONG 0.3 10 0.1
\]

USERBSDF Example 2

Create a simple BSDF model having constant TIS that is Lambertian near normal incidence, and becomes more specular as grazing incidence angles are encountered.

\[
$FCN ROUGH \quad (\frac{4 \times (1 - 2)}{3.14159})
\]

MODEL

\[
\text{USERBSDF } ROUGH 0.9
\]

When used without a $FCN definition, the USERBSDF routine returns the BSDF as a function of wavelength and isotropic-surface scattering variables. The default definition for
**USERBSDF** is the crude APART model for scattering from a random collection of homogeneous spherical particles where:

\[
\begin{align*}
    c &= \text{particle radius divided by wavelength} \\
    c' &= \text{fractional area obscured by particles} \\
    c'' &= \text{real refractive index of particle (default=100; that is, opaque)}
\end{align*}
\]

As mentioned previously, correct usage of this command should ensure that the user-defined function remains an accurate BSDF model at all angles of incidence. Exception occurs when using the **ROUGHNESS MODEL** command. This command requires only a function that models the BRDF at normal incidence (see “BSDFDATA/RAWDATA (Isotropic)”).

**BSDFDATA/RAWDATA (ISOTROPIC)**

```
BSDF [ n ] [ d ] ...options
RAWDATA
    a0 b0
    a b f
    ...
```

The **BSDFDATA/RAWDATA** command allows you to define a scatter model based upon actual measured data. ASAP interpolates, whether directly (isotropic) or indirectly (anisotropic), from user-supplied data entry. For surfaces which obey symmetry and reciprocity, a single normal incidence data set is sufficient. Since no assumptions are made concerning symmetry, a reliable data set should contain values over most of the output hemisphere for most of the input hemisphere.

A typical data set should contain subsets of measured values for several angles of incidence. Each subset is a doublet \((a_0, b_0)\) followed by an arbitrary number of triplets \((a, b, f)\). The doublet indicates the incident direction cosine coordinates or spherical **ANGLES** from and around normal. Subsequent triplets describe the scattered radiation: an ordered pair \((a, b)\) specifying either direction cosine space coordinates or polar and azimuthal **ANGLES** followed by a third entry \(f\) specifying the corresponding measured data value. In-plane data is entered by specifying zero for all values of \(b\). The **BSDFDATA** keyword indicates to ASAP that the data is in the form \(\log(\text{BSDF})\). ASAP linearly interpolates in log amplitude space for coefficients between angles given. The **RAWDATA** keyword indicates that the data is simply the BSDF.

The **RAWDATA** model is a multi-purpose command in ASAP. It can also be used to implement polarization-sensitive coatings with **COATING MODELS**. In the case of
coatings, the \((a_0 b_0)\) doublet is not required. The triplet data format \((a b f)\) represents input angle \((a |b=0|)\) and reflectance or transmittance \((f)\).

**SUM (ISOTROPIC)**

The `SUM` command adds previously defined BSDF models \(k\) and \(k'\) to form a composite model. This command is fully recursive so that one `SUM` model can reference another. For example,

```
MODELS
HARVEY 0.1 -1.5 0.002 ***model 1
HARVEY 0.4 -1.8 0.001 ***model 2
SUM 1 2 ***model 3
LAMBERTIAN .3 ***model 4
SUM 3 4 ***model 5
RETURN
```

Typical usage of the `SUM` command may involve combining a `HARVEY` model with a `LAMBERTIAN` floor to simulate a shiny paint. The combination of two `HARVEY` models can closely simulate some plastic diffusers made from chemically etched molds, as well as contamination levels due to particulates.
BI-DIRECTIONAL SCATTER

Numerous applications require simulation of bi-directional scatter. The optional syntax,

SCATTER MODELS m m'

allows separate scatter models to be assigned for the forward and backward directions, respectively. Note that, whether or not an INTERFACE command is affixed to the scattering surface, ASAP assigns a local SPLIT 1 to that object by default. This local SPLIT setting overrides any global SPLIT setting either preceding or following that object. If you require a higher SPLIT setting at the scattering surface, you must set it locally.
Anisotropic Scatter Models

Objects that have been brushed, marked, scratched, polished in an orderly fashion or diamond-turned, exhibit-enhanced scatter when viewed or illuminated from particular directions. Such surfaces are termed anisotropic or non-isotropic. Their scattering characteristics are not rotationally symmetric at normal incidence, nor are they necessarily symmetric about the plane of incidence.

Orientation of anisotropic models with respect to a desired surface characteristic is implemented in ASAP with an additional `MODEL` command entry X,Y,Z,U,V,W,R,T, or A, indicating the type of asymmetry. X, Y, and Z are the global coordinate directions. For example, X refers to linear markings parallel to the `global` x-axis and the surface normal ç. α is measured from an axis perpendicular to the x-axis and ç, while β is measured perpendicular to both α and ç. See “Figure 11”

U, V, and W are either the parametric directions for `CURVE` objects or the local x, y, and z coordinates of a `SURFACE`. These designations are for use with arbitrarily oriented objects. U then refers to linear markings perpendicular to the `local` x-axis and the surface normal. Recall that the local coordinate origin is located at the entity reference point. Refer to ASAP Help for reference point information on specific entity types. See “Figure 12”.

![Figure 10 Isotropic (a) and anisotropic (b) scattering surfaces](image-url)
Figure 11 Reference directions for anisotropic scatter models

Figure 12 U and V anisotropies
**SCATTERING IN ASAP**

*Types of Scatter Models*

**R, T, and A** refer to radial, angular, or axial anisotropies for quasi-radially symmetric entities such as lenses, edges formed using **SWEEP AXIS**, or parameterized or localized **SURFACE** objects. Apply **R** when describing radial markings like spokes on a wheel as in “Figure 13a”. As with **X,Y,Z,U,V** and **W**, α is measured from an axis perpendicular to the marking and ϑ while β is measured perpendicular to both α and ϑ. **T** is appropriate for concentric markings found on diamond-turned optics as in “Figure 13b” **A** represents axial markings on cylindrical objects shown in “Figure 13c”. Objects rotated or aligned after an anisotropic **MODEL** is applied maintain a fixed relationship between their **MODEL** orientation and local coordinate systems.

![Figure 13](image)

*Figure 13 a) Radial, b) Tangential, and c) Axial asymmetries*

The following **MODELs can be used to represent anisotropic scatter in ASAP:**

- **HARVEY**
- **NONLINEAR**
- **USERBSDF**
- **BSDFDATA/RAWDATA**
- **SUM**
- **VANES**

**NOTE** Only one anisotropy orientation can be used when summing anisotropic models—either the one specified or the one for the first listed anisotropic model.
HARVEY (ANISOTROPIC)

**HARVEY** bs 1 1' [ w ]

The **ELLiptical HARVEY** is the anisotropic version of a modified Harvey model as defined in the previous section. Command syntax is consistent with the three-parameter isotropic Harvey model. An additional entry following the keyword indicates a plane or axis of symmetry. The BSDF of an elliptic Harvey is expressed as

\[
\text{BSDF}_{\text{Elliptical Harvey}} = b f \left( \frac{\alpha - \alpha_0}{\ell}, \frac{\beta - \beta_0}{\ell'} \right)
\]

\[
f(x, y) = \sqrt{1 + x^2 + y^2},
\]

where \(\alpha, \beta\) and \(\alpha_0, \beta_0\) are the scatter and specular direction cosines, respectively, and \(\ell\) and \(\ell'\) are the perpendicular shoulder points of the anisotropic model.

NONLINEAR (ANISOTROPIC)

**NONLINEAR** x p q a b c d e [ p' q' a' b' c' d' e' ]

The general **NONLINEAR** scatter model is a combination of the hong (broad peak) and Harvey (sharp peak) models and, as such, is applicable to both smooth and rough surfaces. The BSDF of an anisotropic **NONLINEAR** scatter model obeys all the appropriate positivity, symmetry, and reciprocity properties, and is defined by the formula,

\[
\text{BSDF} = \sum_{i=1}^{N} \left\{ p_i \left( A^2 + A_0^2 \right) + q_i \left( B^2 + B_0^2 \right) + a_i A_0 b_i B_0 + c_i C_0 + d_i \right\}^e
\]

where

\[
C^2 = 1 - A^2 - B^2 = \cos^2 \theta
\]

\[
C_0^2 = 1 - A_0^2 - B_0^2 = \cos^2 \theta_0
\]

\(\theta\) and \(\theta_0\) are scatter and specular angles, respectively, measured from the surface normal. The \(e\) exponents do not need to be integer or positive. If the quantity in braces \{\} is less than
(<) zero, the term is set to zero. The total number of parameters must be less than or equal (=) to 285; that is, N less than or equal to 40. Five terms or fewer are usually sufficient for most surfaces.

**NOTE:** When $pi = qi$ and $ai = bi$ for each term, this model reduces to the isotropic-surface version.

As with other anisotropic models, the surface anisotropy is aligned with local Alpha or Beta directions. The second entry on the command line (X,Y,Z,U,V,W,R,T,A) specifies the axis of asymmetry as described in the introductory section on “Anisotropic Scatter Models” on page 40.

The anisotropic **NONLINEAR** model can optionally **FIT** measured data to the above BSDF equation. Data must be formatted as described under the **BSDFDATA** command. The parameter $n$ is the number of 7-parameter terms (default 5) or optionally the $e$’s are the starting guesses for the exponents of each term. The **ANGLES** must be included when specifying spherical angle coordinates. The **LOG** option can be used when BSDF data values are provided in common logarithmic form.

**NOTE:** If data that is provided does not cover most of the input and output hemispheres, the fitted model can exhibit unexpected behavior in the regions where data is absent; for example, the model may have a TIS greater than one.

Since the **NONLINEAR** model is not defined in logarithm space (as is the **POLYNOMIAL** model), the fitting algorithm may have a tough time accurately reproducing any BSDF with a high dynamic range. Optionally, the **FRAC**tional error at each data point can be used instead of the absolute error. This has about the same effect as fitting in logarithm space. The nature of an iterative non-linear damped least-squares fit means the algorithm may converge to a local minima rather than a desired global minima. An off-line, global optimization technique (for example, simulated annealing) could be used if the ASAP fit is unsatisfactory.

**VANES (ANISOTROPIC)**

The **VANES** command was developed as a shortcut for approximating baffle vanes. The model is anisotropic by virtue of the geometry it simulates. **VANES** allows you to model a fictitious surface or cone that rests on the tip of the vanes (see “Figure 14”). The vane cavity surfaces (sides, bottoms and tips) are assumed to be Lambertian with individually specified TIS values.

The advantage of **VANES** is better understood by realizing that a good baffle design requires incident light to scatter from at least two surfaces before exiting. Therefore, a baffle
geometry constructed from a tube or cone with individual vanes requires a minimum of two scatter events to model only the baffle, and this may become a limitation (see “Anisotropic Scatter Models” on page 40). Therefore, the VANES command increases ray tracing efficiency by simulating the complex scatter of vanes cavities as a single event. This virtual-surface model has become obsolete with the advent of ever-faster computers.

Figure 14 VANES diagram illustrating model parameters

**Anisotropic Harvey Example 1**
Construct a cylinder having brush marks on its body that are parallel to the cylinder axis and concentric circular brush marks on its end caps. Apply an elliptical Harvey model to the surface.

```plaintext
MODELS
  HARVEY A 1 -0.25 0.012 0.2 "CYLINDER BODY"
  HARVEY T 1 -0.25 0.012 0.2 "CYLINDER CAPS"
RETURN

SURFACE
  TUBE Z -200 2050 200 2050 200
  OBJECT; .1 COLOR 11 'CYL'
  SCATTER MODEL 1

SURFACE
  PLANE Z -200 ELLI 50
  OBJECT; .1 COLOR 11 'CYLCA1'
  SCATTER MODEL 2
RETURN

SURFACE
  PLANE Z 200 ELLI 50
  OBJECT; .1 COLOR 11 'CYLCA2'
  SCATTER MODEL 2
RETURN
```
Harvey Example 2
Place an anisotropic scatter model on a plane and demonstrate the difference between global and local model orientation.

*Figure 15 Cylinder with anisotropic scatter model $A$ on body and $R$ on ends*

Harvey Example 3
Add an anisotropic scatter model to a diffraction grating.

*Figure 16 Differences between global and local model orientation*
Here, the anisotropic Harvey model has its symmetry axis parallel to the grating lines. The **ALL** option on the **LEVEL** command applies the scatter model to each grating order as shown in “Figure 17”

![Scattered rays from grating orders (left), and SPOTS DIR for scattered rays (right).](image)

**Figure 17** Scattered rays from grating orders (left), and SPOTS DIR for scattered rays (right).

**Model Options**

**PLOT OPTION**

The **PLOT** option can be applied to any model definition. The **PLOT** command generates graphs of the BSDF model (logBSDF vs. \( \beta - \beta_0 \), logBSDF vs. \( \Theta \), BSDF vs. \( \Theta \)) and calculates
Total Integrated Scatter (TIS) for up to seven user-defined incident angles $a \ a' \ldots$. TIS values appear on each graph and are written to the Command Output window as shown below.

---PIXELS 1001
---MODELS
--- HARVEY 0.1 -1.25 .002 PLOT 0 30 45 60

<table>
<thead>
<tr>
<th>Angle</th>
<th>Relative TIS</th>
<th>Sampling</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.351E-03</td>
<td>radial</td>
</tr>
<tr>
<td>0.0</td>
<td>0.351E-03</td>
<td>1001x1001</td>
</tr>
<tr>
<td>30.0</td>
<td>0.329E-03</td>
<td>1001x1001</td>
</tr>
<tr>
<td>45.0</td>
<td>0.302E-03</td>
<td>1001x1001</td>
</tr>
<tr>
<td>60.0</td>
<td>0.269E-03</td>
<td>1001x1001</td>
</tr>
</tbody>
</table>

Careful attention should be paid to the PIXELS setting when relying upon the accuracy of these TIS values. Inadequate sampling can lead to inaccuracies in TIS values, particularly in the case of a sharply peaked BSDF function. Remember, ASAP is doing the TIS integral, so keep increasing the PIXEL count until the individual values converge.

When the PLOT option is used, ASAP also creates distribution files ($\text{modelname\_angle.dis}$) that are associated with each incident angle $a \ a' \ldots$. These distribution files can be opened in the DISPLAY mode making any of the File, Graphics, or Processing functions available.

**MINMAX OPTION**

...MINMAX $b \ b'$

The ...MINMAX option overrides the BSDF definition at its extremities with user-defined values. This option provides the user with a convenient method of adjusting a given model to better agree with measured data.

**SCATTER METHODS**

---TOWARDS Command

An importance area is a construct that increases the efficiency and accuracy of scatter calculations. Its purpose is to specify a preferential direction, or limit the solid angle into which rays are traced. The importance area is implemented in ASAP with the TOWARDS command as a modifier to SCATTER MODEL/RANDOM. The TOWARDS command syntax is shown below. Up to 20 TOWARDS modifiers can be used for any given object.
If an **EDGE** or a number follows the **TOWARDS** option, the absolute value of \( i \) is the number of the **EDGE** used as importance areas. If \( i \) is entered as a positive number, radiation is scattered towards the designated real area; if \( i \) is negative, radiation is scattered away from the designated virtual area.

**NOTE** The **TOWARDS** command is designed for use only with **RECTANGLE** or **ELLIPSE** entities. Attempting to scatter toward points-based **EDGES**, such as **POINTS** and **CURVE**, may yield unpredictable results.

The entry \( n' \) is the number of randomly directed rays scattered into the importance area or importance direction by each **TOWARDS** command. The \( f \) and \( t \) are the fractional lower and upper bounds (default 0 to 1) of the importance area relative to the defining edge. ASAP does not take into account importance areas whose solid angles overlap to maintain conservation of energy.
Instead of an EDGEx, an importance direction can be specified as lying between the surface normal and one of the following entries, as illustrated in “TOWARDS command options SPEC, EDGEx, and POINT with \( f=0.1 \)’.

<table>
<thead>
<tr>
<th>Entry</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X,Y,Z,-X,-Y,-Z )</td>
<td>Global coordinate axis direction</td>
</tr>
<tr>
<td>REFL, TRAN, SPEC</td>
<td>Reflected, transmitted, or either specular direction</td>
</tr>
<tr>
<td>POINT ( i )</td>
<td>Direction from ray point to entity &quot;i&quot; reference point</td>
</tr>
</tbody>
</table>

In these cases, \( f \) is the cone half-angle in radians of the solid angle centered on the direction. The \( t \) is a parameter that varies the scattering direction from the surface normal (\( t=0 \)) to the specified direction (\( t=1 \), default). A value of \( t=-1 \) corresponds to the retroREFLection.

Figure 18 TOWARDS command options SPEC, EDGEx, and POINT with \( f=0.1 \)

**Scattered Ray Flux Assignment**

You are free to choose the number of scattered rays \( n' \) per incident ray. During the trace, the ASAP random number generator chooses the direction cosines for each of the \( n' \) scattered rays. For importance EDGExes, the flux assigned to each scattered ray is

\[
F_s = F_0 \ BSDF(\theta_i, \phi_i, \theta_s, \phi_s) \frac{A \cos \theta_i \cos \theta_s}{r_i' R^2}
\]
where $F_s$ is the scattered ray flux, $F_0$ is the incident ray flux, $A$ is the area of the importance edge, $\Theta_c$ is the angle between the scattered ray and the importance edge surface normal, $\Theta_s$ is the angle between the scattered ray and the scattering surface normal, and $R$ is the distance from the scattering surface to the importance edge along the path traveled by the scattered ray. The BSDF value for each ray is calculated from the appropriate model using the incident and scattered ray direction cosines. For the case of importance directions, scattered rays are assigned flux according to the expression

$$F_s = F_0 \cdot \text{BSDF} \left( \theta_i, \phi_i, \theta_s, \phi_s \right) \frac{\Gamma}{n},$$

where $\Gamma$ is the projected solid angle based upon the specified cone angle $\phi$. 
**Signal-to-Noise in a Ray Tracing Calculation**

Signal-to-noise in a ray tracing calculation is governed by Poisson statistics. The implication is that accuracy increases with the number of rays traced. Since the number of rays traced is inexorably tied to CPU time, a prudent choice of $n'$ becomes imperative, especially in light of the numerous scattering objects commonly encountered in stray light analyses. The methods of flux assignment outlined above insure an accurate result by using one scattered ray per incident ray, provided the cone half-angle is greater than 0.1, or in situations where the probability of a ray reaching the target object is reduced by aberration. Such situations inevitably occur when assigning importance **edges** to intermediate optical surfaces within multi-element lenses or telescopes.
Statistics of Ray tracing

As an estimate of the error based entirely on ray statistics, the signal-to-noise ratio is traceable to a Bernoulli trial sequence. The signal-to-noise ratio is statistically defined as the ratio of the distribution mean to the square root of the distribution variance within an energy counting bucket. For a Bernoulli trial sequence, the signal-to-noise ratio is:

\[
\frac{\text{signal}}{\text{noise}} = \frac{\langle m \rangle}{\sigma} = \frac{\sqrt{Np}}{\sqrt{1 - p}} = \frac{\sqrt{n}}{\sqrt{1 - p}}
\]

where:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>square root of the variance = ( \text{SQRT}[Np(1-p)] )</td>
</tr>
<tr>
<td>\langle m \rangle</td>
<td>mean value = ( Np )</td>
</tr>
<tr>
<td>N</td>
<td>total number of rays in ray trace</td>
</tr>
<tr>
<td>p</td>
<td>probability of a ray getting to the detector</td>
</tr>
<tr>
<td>n</td>
<td>number of rays at the detector</td>
</tr>
</tbody>
</table>

In the case of a small probability of a ray getting to the detector; that is, \( p \ll 1 \), the statistical error reduces to the mean and variances of a Poisson distribution. The Poisson distribution is the limiting case of the Bernoulli trial sequence under this condition. The noise-to-signal ratio becomes:

\[
\frac{\text{signal}}{\text{noise}} = \sqrt{n}
\]

When other cascading effects such as source apodization and system transmission are accounted for, the noise-to-signal ratio in an energy-counting bucket becomes:

\[
\frac{\text{signal}}{\text{noise}} = \sqrt[2]{\frac{\phi_{\text{total}}}{\phi_{\text{max}}}}
\]

where:

- \( \phi_{\text{max}} \) = maximum flux of ray in energy counting bucket
- \( \phi_{\text{total}} \) = total flux of rays in energy counting bucket
Energy Conservation

Conservation of energy can be described by a simple relationship involving specular, absorptive and scatter components of a field:

\[ R + T + A + S = 1 \]

The sum of reflectance, transmittance, absorptance and total integrated scatter equals unity. This is the natural state of affairs in our everyday world; a condition we take for granted in any situation.

**NOTE** Even though ASAP is a highly refined tool for simulating a broad range of opto-mechanical phenomena, it is simulation none the less. The accuracy of any simulation is ultimately limited only by the available tools and ingenuity of the user. While ASAP is undoubtedly the most versatile and powerful optics analysis program, one of the few caveats is that ASAP does not guarantee energy conservation in all cases involving scatter. By design, the specular and absorptive properties set by the **INTERFACE** command are decoupled from properties assigned by the aforementioned scatter designators, **SCATTER RANDOM/MODELS** and **ROUGHNESS**.

Only one case exists in which energy conservation can be assured even though specular and scatter processes are decoupled. This case arises from the fact that a Lambertian scatter model, unlike any other scatter model type, has a constant TIS that is independent of incident angle. Energy conservation for a Lambertian scatter surface then becomes a simple matter of satisfying the equation by proper choice of a **COATING MODEL** and absorption coefficient.

Importance Area Sampling

Importance **EDGES** are used to aim scattered rays. Ray trace efficiency is increased dramatically when rays are traced only in directions of interest; that is, towards the detector in the case of Point Source Transmittance (PST) calculations, or towards object space when computing system BSDF. An importance edge is determined in the following manner:

1. Fill the system stop with an appropriately shaped **GRID** of rays. Use the commands **SOURCE FOCUS** and **FOCUS** to determine the apparent position of an on-axis point source at the final image surface. Having determined the proper axial position, change **SOURCE FOCUS** to **SOURCE POS** to generate the source to be traced for the remaining steps. This grid helps locate the detector position as seen from each relevant surface.

2. Trace this grid to the optical surface of interest using the **TRACE STEP -n** command. Consider only that surface (use the **CONSIDER ONLY** command with the name of the object of interest) and issue the **FOCUS** command. The axial value produced by the **FOCUS** command gives the position of the detector as viewed from the surface of interest. Record this value for use in step 3.
3 Place an **EMITTING RECT** at the final image plane, equal in dimension to the photosensitive area. Use the \( a^\prime \) option to limit the emission angle to a cone that slightly overfills the stop. Trace this emitter out to the surface of interest using **TRACE STEP** \(-n\). Consider only that surface and issue a **FOCUS MOVE** \( z \), where \( z \) is the axial value determined in step 2 above. Issue a **WINDOW** command followed by **SPOTS POSITION**. The lateral dimensions of the spot diagram give the dimensions of the importance **EDGE**.

4 Repeat steps 2 and 3 for each surface of interest.

**NOTE** In most cases, the spots diagram produced in step 3 has well-defined edges. However, in certain situations where the image is severely aberrated, the spot diagram has a region of high ray density, surrounded by a larger area of lower ray density. You may choose to use the entire area of the spot diagram; however, using only the area of high ray density leads to a more efficient ray trace, while introducing negligible error in most cases.

**MONTECARLO Technique**

**SPLIT n MONTECARLO**

With the **MONTECARLO** option, no additional specular and/or scattered rays are created. Both the total number of rays and the total power are conserved. After intersecting an object, the direction of each ray is selected randomly from among the assigned reflection, refraction, diffraction, or scatter properties. The probability that a given direction is selected is proportional to the power going in that direction.

**NOTE** Be aware of the statistical nature of the **MONTECARLO** option. An estimate for the number of rays required to achieve a desired accuracy should be obtained by applying Poisson statistics.

The following figure illustrates the difference between conventional ray scattering (left) and scattering (right) with the **MONTECARLO** option.

**Figure 19 Example of scatter using conventional ray splitting and Monte Carlo analysis**
A ray with a power of 1 is incident upon a surface that scatters 40% of light back from the surface and 50% of the light through the surface. The BSDF is Lambertian in both directions. Conventional ray scattering, shown on the left half of the figure, generates a child ray with a power of 0.4 in a random direction into the backward hemisphere. The transmitted parent ray has its power reduced to 0.5, and it is given a random direction into the forward hemisphere. The total power is not conserved (because absorbed power is not accounted for), and the number of rays has doubled. The right half of the figure illustrates scatter with the MONTECARLO option. Again, a ray with a power of 1 is incident on the surface, but this time no new rays are generated; instead, the incident ray is reused. The ray is directed into the backward hemisphere with a 40% probability, it is directed into the forward hemisphere with a 50% probability, and it stops on the surface with a 10% probability. The total power is conserved and the number of rays is not changed.

If the total power from reflection, refraction, diffraction, and scatter is less than 1 on an object, the ray may stop on that object. This represents absorption by that object.

**CAUTION** When the MONTECARLO option is selected, you must arrange the interface and scatter commands so the total power from reflection, refraction, diffraction, and scatter never exceeds 1 for any angle of incidence. Failure to do this causes errors in the power calculations.

A summary is given here of how scatter is modeled by ASAP when the SPLIT MONTECARLO option is used in combination with SCATTER MODEL and TOWARDS commands. When a ray strikes a scattering surface, ASAP generates a random number between 0 and 1; call it $r$. ASAP then generates a random direction and calculates a power that is given by

$$P_i = \rho(\mathbf{u}, \mathbf{u}_0) \frac{\pi}{N},$$

where $P$ is the BRDF for the random scatter direction $\mathbf{u}$ and the specular direction $\mathbf{u}_0$, and $N$ is the entry for the number of scattered rays on the TOWARDS command. If the power $P_1$ is greater than random number $r$, the ray that struck the surface is traced in the direction $\mathbf{u}$. If the power $P_1$ is less than $r$, ASAP makes another trial. The random number $r$ is unchanged, but a new direction is generated, and a new power, $P_2$ is calculated.

ASAP compares the sum $P_1 + P_2$ to the random number $r$. If the sum is greater than $r$, the ray is traced in the new scatter direction. If the sum is less than $r$, ASAP retries. This process is repeated $N$ times (the entry for the number of rays on the TOWARDS command).

If after all $N$ trials, the sum $P_1 + P_2 \ldots + P_N$ is still less than $r$, the ray sticks to the surface; that is, the ray is not traced further.
NOTE The power of the ray is not changed, regardless of whether it is traced further or not. Also, no new rays are generated, regardless of the values for N. Essentially, N is the maximum number of trials for the scatter direction.

When the TOWARDS command is used to simulate scatter from an object, the following rules should be obeyed:

1. If an object has a Lambertian scatter model attached to it, the entry for the number of rays should be 1. Entering a larger value only slows down the ray trace without improving the accuracy of the calculation.

2. For objects that have a non-Lambertian scatter model, the accuracy of the calculation improves as the entry for the number of rays gets larger.

3. For non-Lambertian BRDFs, if the maximum BRDF over all relevant angles of incidence and scatter is $P_{\text{max}}$, the number of scattered arrays $N$ on the TOWARDS command should be greater than $\pi P_{\text{max}}$.

4. In most cases, the TOWARDS SPEC option should be used with scatter into the hemisphere above an object: TOWARDS SPEC (N) 3.14/2 0, where $N$ is the entry for the number of scattered rays.

You can test these ideas with the script in Figure 20. A BRDF is defined that is constant inside a cone centered on the surface normal. If the variable $N$ is greater than or equal to RHO, in keeping with rule 3 above, the total integrated scatter that is calculated by the ray trace is correct. If $N$ is less than RHO, the total integrated scatter is wrong.
Users may combine reflection, transmission, and scatter on an object when using the MONTECARLO option. The INTERFACE, SCATTER, and TOWARDS commands are used in the usual way, except the entry for the number of scattered rays is interpreted as the number
of random trials for a scatter direction (as discussed above). Incident rays are directed in a particular specular or scattered direction (or stop on the object when absorbed) with probabilities that are proportional to the power going in each direction.

**NOTE** The total of reflection, transmission, and scatter must not exceed 1 for all angles of incidence.

If desired, the **MONTECARLO** option may be applied to only selected objects. This is done in the same way as for the conventional **SPLIT** and **LEVEL** commands. When the **SPLIT MONTECARLO** command is entered below the **OBJECT** command (or embedded within or below other commands that refer to that object), only that object is affected. This is illustrated in the following example:

```
SURFACE
   PLANE 2 0 REC 1
OBJECT 'DIFFUSER'
   LEVEL 1
   SPLIT 2 MONTECARLO
INTERFACE .05
SCATTER RANDOM 1.0 1 ABS .4 .5
RETURN
```

*Figure 21 Example script for SPLIT MONTECARLO command*

When a **SPLIT MONTECARLO** command is used an extra split sometimes is displayed in the path table that is produced by the **PATH** command. The script in Figure 20 produces the following table when a **PATH** command is executed.

```
<table>
<thead>
<tr>
<th>Path</th>
<th>Rays</th>
<th>SumTOTA</th>
<th>Percent</th>
<th>Hits</th>
<th>Curr</th>
<th>Prev</th>
<th>Split/Scatter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4944</td>
<td>4.94E-01</td>
<td>49.44</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1.000 0.000</td>
</tr>
<tr>
<td>2</td>
<td>4002</td>
<td>4.00E-01</td>
<td>40.02</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1.000 1.100 0.000</td>
</tr>
<tr>
<td>3</td>
<td>1054</td>
<td>1.05E-01</td>
<td>10.54</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.000</td>
</tr>
</tbody>
</table>

An extra split was added to this scatter path.
```

*Figure 22 Example script for extra split in scatter path*
The extra split, indicated by the 1.100 entry on Path 2, appears to the right of the scatter entry, -1.000. Even though no actual specular split occurs, you should expect to see these extraneous entries in the path table whenever a MONTECARLO is used.

The number of generations entered on the SPLIT command should also be adjusted upward to allow for the presence of these fictitious splits. In the above example, SPLIT should be followed by a value of at least 1, even though no actual specular splits occur, to allow for the extraneous split.

When the total integrated scatter is much less than 1, as it is for black surfaces or mirrors and lenses, most of the incident rays are not scattered. For most illumination systems, this is acceptable, and even desirable. However, if you are doing stray light analysis, you are most interested in low levels of scattered light and specular ghosts. In this case, it is faster and more accurate to use conventional ray splitting rather than the MONTECARLO option.

**BSDF FIT UTILITY**

ASAP offers Harvey and Polynomial models in an interactive BSDF Fit utility, as shown in “Figure 23” This feature is accessed by right-clicking BSDF Fit Utility in the ASAP Workspace to start a New fit or Open an existing file. You are free to vary model input parameters to achieve the desired fit, after which a single operation copies the proper model syntax to the Clipboard for easy pasting into an INR file.
The BSDF Fit Utility supports the following file types:

- ASTM (American Society for Testing and Materials), as outlined in the document “Standard Practices for Angle Resolved Optical Scatter Measurements on Specular or Diffuse Surfaces.”

- ASCII-formatted output generated by Schmitt Industries.

- Harvey Fit files (*.hs) originally saved under ASAP 6.6.

- Any ASCII file created with the following format:

```
ao    a    f
```

where \( a_o \) is the angle of incidence in degrees measured from the surface normal, \( a \) is the scatter angle in degrees also measured from the specular direction, and \( f \) is the measured BSDF value. In all cases, you can correctly import only in-plane data with a fixed angle of incidence. Once a file is imported, the data is displayed in the graphs entitled Measured Forward BSDF and Measured Backward BSDF. The Harvey or Polynomial model can now be fit to the measured data. Saving the file in the BSDF Fit utility (File > Save) creates a binary file with the extension *.fit, which contains all measured data, parameters, and any other settings (such as view size). Your original data files remain unchanged.
SCATTERING IN ASAP

BSDF Fit Utility